**STUDY OF A MANUFACTURING FACILITY**

**SYSC 5001**

**Project Deliverable 2**

**Advisor:**

Prof. Chang Cheng Huang

Dept. of Systems and Computer Engineering

Carleton University

**Group 101:**

101206784 Abdul Mutakabbir

# Identify Distribution (Histogram):

Below will be histograms for the data sets provided.

Chart, histogram

Description automatically generated Chart, histogram

Description automatically generated Chart, histogram

Description automatically generatedChart, histogram

Description automatically generatedChart, histogram

Description automatically generated Chart, histogram

Description automatically generated

It can be seen form the distributions that all the datasets follow exponential distribution.

# Evaluate Distributions (Q-Q Plots):

Chart

Description automatically generatedIn this section the Q-Q plots will be provided based on which the evaluation of the data will be done.

Based on the Q-Q Plot for the exponential distribution of the Workstation 1’s Processing Time, we can say that “Exponential distribution” is approximately a good distribution as it mostly follows a straight line start to middle and start. Later for higher values at the ends it diverges.

Chart, line chart

Description automatically generated Based on the Q-Q Plot for the exponential distribution of the Workstation 2’s Processing Time, we can say that “Exponential distribution” is not a very good distribution overall. This is because it only follows a straight line at the starting quantiles and later diverges from the straight line slightly in the middle and extremely towards the end. But still is a good fit.

Chart, line chart

Description automatically generatedBased on the Q-Q Plot for the exponential distribution of the Workstation 3’s Processing Time, we can say that “Exponential distribution” is a very good distribution overall. It is a better fit that WS1. It follows a straight-line form start to end and barely deviates.

Chart

Description automatically generatedBased on the Q-Q Plot for Exponential distribution of Inspector 1’s Service Time for Component 1 we can say it follows “Exponential distribution” as the points are mostly on straight line at the start, middle and end of the distribution.

Similarly for Inspectors 2’s Service Times for Component 2 and 3, the distribution follows “Exponential Distribution” pattern as the points are mostly on a straight line at the start and middle of the distribution. For component 3 it is a much better fit as it is only slight deviating at the end while the rest is on a straight line.

Chart

Description automatically generatedChart, line chart

Description automatically generated

# Goodness of Fit (Chi-Square Test):

## Workstation 1 Processing Time:

H0: Random variable is exponential distribution.

H1: Random variable is not exponential distribution.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | range | O | O (new) | E | E (new) | O-E | (O-E)^2/E |
| 0 | (0, 4] | 183 | 183 | 174.155 | 174.155 | 8.845047 | 0.449226 |
| 1 | (4, 8] | 64 | 64 | 73.05513 | 73.05513 | -9.05513 | 1.122376 |
| 2 | (8, 12] | 29 | 29 | 30.64542 | 30.64542 | -1.64542 | 0.088346 |
| 3 | (12, 16] | 12 | 12 | 12.85525 | 12.85525 | -0.85525 | 0.056899 |
| 4 | (16, 20] | 7 | 12 | 5.392564 | 9.00161 | 2.99838 | 0.998742 |
| 5 | (20, 24] | 3 | 2.262092 |
| 6 | (24, 28] | 1 | 0.94891 |
| 7 | (28, 32] | 1 | 0.398052 |
| Sum: | | | | | | | 2.715589 |

Degrees of freedom = k – s – 1 = 5 – 1 – 1 = 3

Level of significance = 0.05

Chi-Square (0.05,3) = 7.81

2.71 < 7.81

**H0 is accepted**

## Workstation 2 Processing Time:

H0: Random variable is exponential distribution.

H1: Random variable is not exponential distribution.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | range | O | O (New) | E | E(New) | O-E | (O-E)^2/E |
| 0 | (0, 7] | 147 | 147 | 140.3903 | 140.3903 | 6.60971 | 0.311192 |
| 1 | (7, 14] | 76 | 76 | 74.69218 | 74.69218 | 1.307822 | 0.022899 |
| 2 | (14, 21] | 29 | 29 | 39.73866 | 39.73866 | -10.7387 | 2.901929 |
| 3 | (21, 28] | 19 | 19 | 21.14225 | 21.14225 | -2.14225 | 0.217065 |
| 4 | (28, 35] | 9 | 9 | 11.24836 | 11.24836 | -2.24836 | 0.44941 |
| 5 | (35, 42] | 9 | 9 | 5.984493 | 5.984493 | 3.015507 | 1.519475 |
| 6 | (42, 49] | 5 | 11 | 3.183944 | 5.779147 | 5.220853 | 4.716492 |
| 7 | (49, 56] | 5 | 1.693961 |
| 8 | (56, 63] | 1 | 0.901242 |
| Sum: | | | | | | | 10.138462 |

Degrees of freedom = k – s – 1 = 7 – 1 – 1 = 5

Level of significance = 0.05

Chi-Square (0.05,3) = 11.1

10.13 < 11.1

**H0 is accepted**

## Workstation 3 Processing Time:

H0: Random variable is exponential distribution.

H1: Random variable is not exponential distribution.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | range | O | O (New) | E | E (New) | O-E | (O-E)^2/E |
| 0 | (0, 6] | 155 | 155 | 148.343 | 148.343 | 6.657041 | 0.298741 |
| 1 | (6, 12] | 70 | 70 | 74.99085 | 74.99085 | -4.99085 | 0.332155 |
| 2 | (12, 18] | 34 | 34 | 37.90963 | 37.90963 | -3.90963 | 0.403202 |
| 3 | (18, 24] | 23 | 23 | 19.16421 | 19.16421 | 3.835791 | 0.767748 |
| 4 | (24, 30] | 7 | 7 | 9.687958 | 9.687958 | -2.68796 | 0.745783 |
| 5 | (30, 36] | 7 | 11 | 4.89749 | 9.257559 | 1.742441 | 0.327959 |
| 6 | (36, 42] | 2 | 2.475796 |
| 7 | (42, 48] | 1 | 1.251573 |
| 8 | (48, 54] | 1 | 0.6327 |
| Sum: | | | | | | | 2.875588 |

Degrees of freedom = k – s – 1 = 6 – 1 – 1 = 4

Level of significance = 0.05

Chi-Square (0.05,3) = 9.49

2.87< 9.49

**H0 is accepted**

## Inspector 1 Service Time for Component 1:

H0: Random variable is exponential distribution.

H1: Random variable is not exponential distribution.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | range | O | O (New) | E | E (New) | O-E | (O-E)^2/E |
| 0 | (0, 11] | 196 | 196 | 196.2699 | 196.2699 | -0.26992 | 0.000371 |
| 1 | (11, 22] | 70 | 70 | 67.86365 | 67.86365 | 2.136351 | 0.067252 |
| 2 | (22, 33] | 24 | 24 | 23.46501 | 23.46501 | 0.534993 | 0.012198 |
| 3 | (33, 44] | 7 | 10 | 8.113424 | 12.22417 | 2.22417 | 0. 404684 |
| 4 | (44, 55] | 1 | 2.805354 |
| 5 | (55, 66] | 1 | 0.969999 |
| 6 | (66, 77] | 1 | 0.335393 |
| Sum: | | | | | | | 0.484505 |

Degrees of freedom = k – s – 1 = 4 – 1 – 1 = 2

Level of significance = 0.05

Chi-Square (0.05,3) = 5.99

0.48 < 5.99

**H0 is accepted**

## Inspector 2 Service Time for Component 2:

H0: Random variable is exponential distribution.

H1: Random variable is not exponential distribution.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | range | O | O (New) | E | E (New) | O-E | (O-E)^2/E |
| 0 | (0, 17] | 198 | 198 | 199.5547 | 199.5547 | -1.5547 | 0.012112 |
| 1 | (17, 34] | 74 | 74 | 66.81444 | 66.81444 | 7.18556 | 0.772771 |
| 2 | (34, 51] | 20 | 20 | 22.37066 | 22.37066 | -2.37066 | 0.251222 |
| 3 | (51, 68] | 5 | 8 | 7.490091 | 11.1187 | 3.1187 | 0.874768 |
| 4 | (68, 85] | 1 | 2.507815 |
| 5 | (85, 102] | 1 | 0.839661 |
| 6 | (102, 119] | 1 | 0.281133 |
| Sum: | | | | | | | 1.910873 |

Degrees of freedom = k – s – 1 = 4 – 1 – 1 = 2

Level of significance = 0.05

Chi-Square (0.05,3) = 5.99

1.91 < 5.99

**H0 is accepted**

## Inspector 2 Service Time for Component 2:

H0: Random variable is exponential distribution.

H1: Random variable is not exponential distribution.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | range | O | O (New) | E | E (New) | O-E | (O-E)^2/E |
| 0 | (0, 15] | 147 | 147 | 154.9928 | 154.9928 | -7.99283 | 0.412183 |
| 1 | (15, 30] | 85 | 85 | 74.91691 | 74.91691 | 10.08309 | 1.357088 |
| 2 | (30, 45] | 34 | 34 | 36.21163 | 36.21163 | -2.21163 | 0.135075 |
| 3 | (45, 60] | 18 | 18 | 17.50315 | 17.50315 | 0.496849 | 0.014104 |
| 4 | (60, 75] | 7 | 7 | 8.460275 | 8.460275 | -1.46027 | 0.252049 |
| 5 | (75, 90] | 5 | 9 | 4.089335 | 6.065944 | 2.934056 | 1.419183 |
| 6 | (90, 105] | 4 | 1.976609 |
| Sum: | | | | | | | 3.589682 |

Degrees of freedom = k – s – 1 = 6 – 1 – 1 = 4

Level of significance = 0.05

Chi-Square (0.05,3) = 9.49

3.58 < 9.49

**H0 is accepted**

Based on the Chi-Square Test conducted for all 6 Datasets we can say that all 6 follow Chi-Square Distribution.

# Generate Input Based on Model:

Model Identified: Exponential Distribution

Parameters need: λ

Parameter estimate: λ = 1/µ

## Procedure:

Step 1:

Generate uniform distribution of Sudo Random Number by Linear Congruential Method (Refer 7.3.1 in Textbook).

Xi+1 = (a Xi + c) mod m i = 1, 2, 3, 4, …

Here follow best practices mentioned in the textbook.

m -> a power of 2, as large as possible

* m = 248

c -> not equal to zero, relatively prime to m

* c = 27 – 1

a -> 1 + 4k where k is integer

* a = 1 + 4 \* 2 = 9

X0 any arbitrary value

Step 2:

Convert Xi to the range [0, 1] resulting in a uniform distribution.

Ri = Xi / m

Step 3:

Use this random uniform distribution of numbers to get exponential distribution with the following formula (detailed explanation in 8.1.1 Textbook):

Xi = - ln(Ri) / λ

(or)

Xi = - ln(Ri) \* µ

Step 4:

Generate the numbers

## Resultant Distribution:

Based on the above-mentioned steps a generator was made.

The output of distribution for Step2, 3 are presented for the generator for the Mean = 8.45

Chart, histogram

Description automatically generatedChart, histogram

Description automatically generated

## Frequency Test (K-S Test):

H0: Random variable is Uniform [0,1]

H1: Random variable is not Uniform [0,1]

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Range | O (Observed) | T (Theoretical) | FO (X) | FT(X) | | FO (X) - FT(X)| |
| (0.0, 0.1] | 1018 | 1000 | 1018/10000 | 1000/10000 | 18/10000 |
| (0.1, 0.2] | 1006 | 1000 | 1006/10000 | 1000/10000 | 6/10000 |
| (0.2, 0.3] | 997 | 1000 | 997/10000 | 1000/10000 | 3/10000 |
| (0.3, 0.4] | 1035 | 1000 | 1035/10000 | 1000/10000 | 35/10000 |
| (0.4, 0.5] | 939 | 1000 | 939/10000 | 1000/10000 | 61/10000 |
| (0.5, 0.6] | 961 | 1000 | 961/10000 | 1000/10000 | 39/10000 |
| (0.6, 0.7] | 1040 | 1000 | 1040/10000 | 1000/10000 | 40/10000 |
| (0.7, 0.8] | 990 | 1000 | 990/10000 | 1000/10000 | 10/10000 |
| (0.8, 0.9] | 987 | 1000 | 987/10000 | 1000/10000 | 13/10000 |
| (0.9, 1.0] | 1027 | 1000 | 1027/10000 | 1000/10000 | 27/10000 |

N = 10000

α = 0.05

D = max (| FO (X) - FT(X)|)

* D = 61/10000 = 0.0061

D10000 0.05 = 1.36/Sqrt(10000) = 1.36/100 = 0.0136

0.0061 < 0.0136

**H0 is accepted**

## Test for Autocorrelation:

First 10 random numbers are:

[0.33, 0.04, 0.40, 0.60, 0.44, 0.02, 0.22, 0.01, 0.12, 0.10]

H0: ρim = 0 if numbers are independent

H1: ρim != 0 if numbers are dependent

l = 1

α = 0.05

N = 10

i = 1 (start from the first number)

M = 8

* i + (M +1) l <= N
* 1 + (M + 1) 1 <= 10
* M + 2 <= 10
* M <= 8

ρ11 = 1/(M +1) [Sum0, M(Ri+kl\*Ri+(k+1)l)] – 0.25

= 1/(8+1) [0.33\*0.04 + 0.04\*0.40 + 0.40\*0.60 + 0.60\*0.44 + 0.44\*0.02 + 0.02\*0.22 + 0.22\*0.01 + 0.01\*0.12] – 0.25

= 1/9 \*[0.5498] – 0.25

= -0.188

σ ρ11 = Sqrt(13 \* M + 7)/ (12 \* (M + 1))

= Sqrt(13 \* 8 + 7) / (12 \* (8 + 1))

= Sqrt(111) / (108)

= 10.535 /108

= 0.097

Z0 = - 0.188/0.097 = -1.947

Z0.025 = 1.96

-1.947 < 1.96

**H0 is accepted**

## Code:

class Random:  
  
 \_\_seed = None  
 \_\_multiplier = 9  
 \_\_increment = 2\*\*7 - 1  
 \_\_modulus = 2\*\*48  
 \_\_last\_rand = None  
  
 # Constructor  
 def \_\_init\_\_(self, seed=0):  
 self.seed = seed  
 self.last\_rand = seed  
  
 # Returns uniform distribution of random number  
 def random\_probability(self):  
 self.last\_rand = (self.\_\_multiplier \* self.last\_rand + self.\_\_increment) % self.\_\_modulus  
 return self.last\_rand / self.\_\_modulus

# Returns randomly distributed exponential numbers  
 def random\_exponential(self, mean=None):  
 if (mean is None) or (math.isnan(mean)):  
 raise Exception("Mean Not Specified")  
 uniform\_rand = self.random\_probability()  
 return -1 \* mean \* math.log(uniform\_rand, math.e)